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We start with the equation

$$\frac{2^x + 10}{4} = \frac{9}{x^{x-2}},$$

and our goal is to find the sum of all real solutions x .

Step 1. Clear the Denominators

Multiply both sides by $4x^{x-2}$ (noting that for a real solution we must have $x > 0$ so that the expression is defined):

$$(2^x + 10)x^{x-2} = 36.$$

Step 2. Analyze the Function

Define

$$f(x) = (2^x + 10)x^{x-2}.$$

We want the values of x for which $f(x) = 36$.

Behavior as $x \rightarrow 0^+$

When $x \rightarrow 0^+$, note that

$$x^{x-2} = x^x \cdot x^{-2}.$$

Since $x^x \rightarrow 1$ as $x \rightarrow 0^+$ and $x^{-2} \rightarrow +\infty$, we have

$$f(x) \rightarrow (2^x + 10) \cdot \infty \rightarrow +\infty.$$

Behavior at $x = 1$ and $x = 2$

- At $x = 1$:

$$\begin{aligned} - 2^1 + 10 &= 2 + 10 = 12, \\ - 1^{1-2} &= 1^{-1} = 1, \\ - \text{ so } f(1) &= 12. \end{aligned}$$

- At $x = 2$:

$$\begin{aligned} - 2^2 + 10 &= 4 + 10 = 14, \\ - 2^{2-2} &= 2^0 = 1, \\ - \text{ so } f(2) &= 14. \end{aligned}$$

Thus, as x increases from 0^+ to 1, $f(x)$ drops from $+\infty$ to 12. Then from $x = 1$ to higher values, $f(x)$ eventually increases again (as the factor 2^x and the rising exponent x^{x-2} take over).

Since

- $f(x) \rightarrow +\infty$ as $x \rightarrow 0^+$,
- $f(1) = 12 < 36$,
- and for larger x (for example, $f(3) = (2^3 + 10) \cdot 3^1 = 18 \cdot 3 = 54 > 36$,

by the Intermediate Value Theorem there must be **two** solutions: one in $(0, 1)$ and another in $(1, 3)$.

Step 3. Find Approximate Solutions

Using numerical methods (or a calculator) one finds:

- **First solution:** $x \approx 0.471$
- **Second solution:** $x \approx 2.755$

Step 4. Sum the Solutions

The sum of the two solutions is approximately

$$0.471 + 2.755 \approx 3.226.$$

Rounded to two decimal places, the sum is about **3.23**.

Final Answer

The sum of all solutions of the given equation is approximately **3.23**.
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